



The Meaning and Use of S-Curves in Cost Estimating

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- **Summary**



“Point” Cost Estimates

- **Project Managers Need “Point Estimates” for ...**
 - ... Cost/Performance Tradeoff Studies
 - ... Cost/Benefit Analyses
 - ... Budget Planning
- **But Project Cost is Often Nebulous due to ...**
 - ... Immature Technology – Too Low TRL
 - ... Uncertain Product Design
 - ... Software-Associated Issues
 - ... Requirements Volatility
 - ... Programmatic and Organizational Considerations
 - ... Schedule Slips due to Integration Difficulties and Test Failures
 - ... Unforeseen Events
- **While “Point” Estimates are not “Correct”, “Actual” Program Cost Will Typically Fall within Some Range (with some degree of confidence)**
 - The Best We Can Hope to Do Is to Understand the Uncertainty
 - Understanding the Uncertainty Will Help Us Make Provision for It



What is a “Point” Estimate?

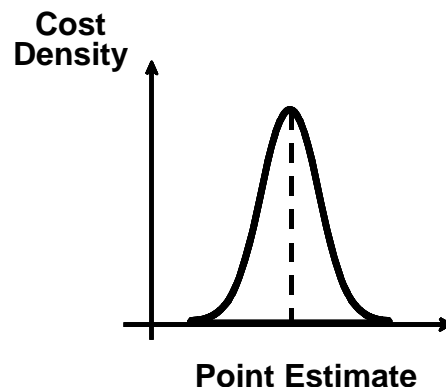
- **Is the “Point” Estimate Equal to ...**
 - ... the “Most Likely” or “Most Probable” Cost?
 - ... the 50th-Percentile Cost?
 - ... the Expected Cost?
- **No – It is not Likely to be Equal to Any of These**
- **There is, in fact, a Range of Possible Cost Values**
 - The “Point” Estimate is not the Only Possible Estimate
 - If the “Point” Estimate is the “Most Likely” Cost, then Other Cost Levels Can be Assumed to be “Less Likely”
 - If the “Point” Estimate is the “50th Percentile” Cost, then There are Cost Levels Corresponding to 99 Other Percentiles
 - If the “Point” Estimate is the “Expected” Cost, then Other Cost Levels are Presumably “Unexpected”



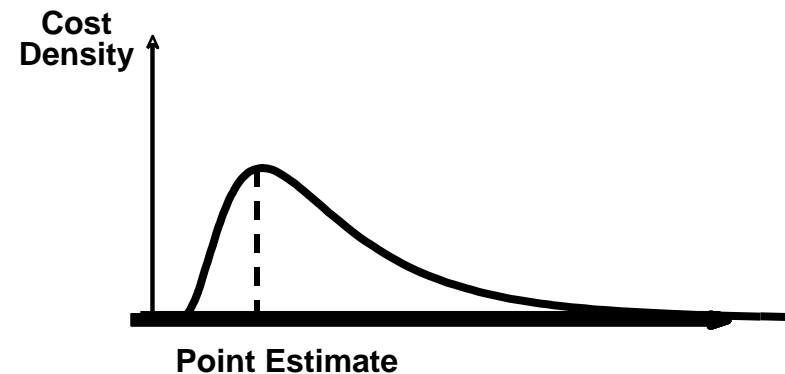
Determining the Range of Possible Cost Levels

- **Information Needed to Estimate Project Cost**
 - Technical Description of Project (e.g., CADRe)
 - Risk Watch List and Management Plan
- **Risks, Technical and Otherwise, Drive the Range of Possible Estimates for Each WBS Elements**

Cost Probability Distribution for Low-Risk Cost Element



Cost Probability Distribution for High-Risk Cost Element





Examples of Cost Drivers

- **Product Capabilities**
- **Component, Subsystem, Assembly Weights**
- **Number of Software Lines of Code**
- **Data-Processing Requirements**
- **Power Requirements**
- **Signal Strength**
- **Special Equipment Needs**
- **Etc., etc., etc.**



Some Technical Risk Drivers

- **Beyond-State-of-the-Art Technology Needs**
 - Data-Processing Capability
 - Communication Links
 - Temperature and other Environmental Conditions
 - Power Requirements
 - Software Solutions
- **Tight Schedules for ...**
 - Technology Development
 - Software Development and Testing
 - Integration and Testing of COTS Software



Some Programmatic Risk Drivers

- **System Integration**
 - Multicontractor Teams and Organizational Interfaces
 - Conflicting Schedules and Workload
 - System Testing and Retesting
- **Special Circumstances**
 - Geographic Distribution of Production Sites
 - Security Arrangements
 - Procurement Quantities
- **Limited Resources**
 - Production Funding
 - Trained Personnel
 - Program Funding Stretch-Out
- **Supplier Viability**



Some Cost-Estimating Risk Drivers

- **Statistical Character of Cost-Estimating Relationships (CERs)**
 - Standard Error of the Estimate
 - Bias
- **Analogies Pushed Beyond Realistic Limits**
- **Estimates Inconsistent with Historical Records**
 - Hardware: Dollars per Pound
 - Software: Lines of Code per Developer-Month
 - Communication Speed
 - Data Throughput
 - Staffing, Labor Needs and Costs
 - Maintenance Costs



Risk Alters the Estimating Process

- **It Doesn't Suffice to Represent a WBS Element's Cost as a Single Number**
 - The Presence of Risk Makes any One Number Suspect
 - Depending on the Outcome of the Risk Management Process, that Number May be Too Large or Too Small
- **Risk is Better Accounted for by "Three-Point" Estimates**
 - Most Optimistic Cost (usually denoted *L* for "low")
 - Most Likely Cost (usually denoted *M* for "mode")
 - Most Pessimistic Cost (usually denoted *H* for "high")
- **These Three Estimates are Then Applied to Form a "Triangular" Probability Distribution of Each WBS Element's Cost**



The Statistical Nature of Risk

- **We Must Treat Risk-Associated Costs as Random Variables with Probability Distributions because Not All Risks Will Come to Pass**
- **Neither *L*, *M*, nor *H* Will be the Exact Cost of the WBS Element to which They Correspond**
 - **Uncertainty in How Development and Production Will Actually Proceed Makes it Impossible to Forecast Costs with High Confidence**
 - **Statistical Modeling of the Cost of Each WBS Element Reflects This Uncertainty**
 - **Statistical Summing of WBS-Element Costs (by Monte Carlo sampling or other methods) Allows Us to Obtain a Better Understanding of Total Cost than We Have of Any Particular WBS Element's Cost**

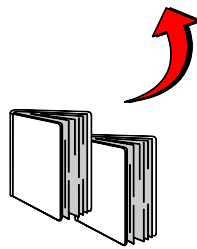
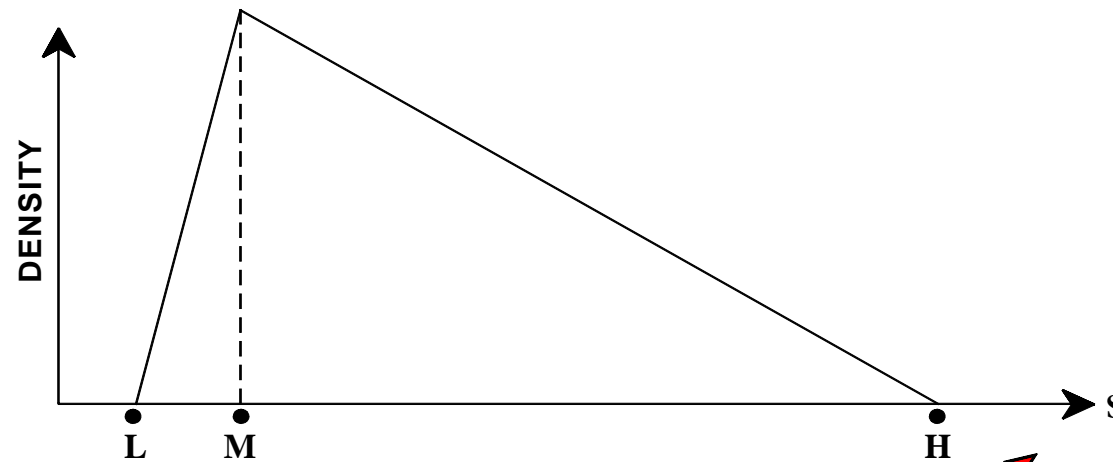


Another Benefit of Statistical Modeling of Costs

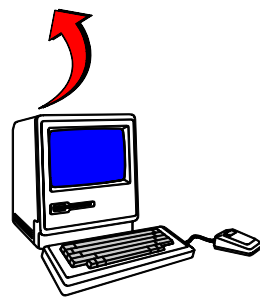
- **Risks are Correlated**
 - Difficulties in Resolving Risk Issues in One WBS Element Often Result in “Fixes” that Impact Other Elements
 - Spending More Money to Resolve One WBS Element’s Risk Issues Often Forces Additional Spending on Other Elements
- **Examples**
 - Technical Risks in a Sensor Subsystem that Lead to Weight Growth or Redesign Tend to Induce Weight Growth or Redesign (and therefore) Cost Growth in Structure, Power, Software, and Other Subsystems
 - Schedule Slippage in One WBS Element Leads to Cost Growth in Other Elements (“standing army effect”)
 - Late-Discovered Hardware Problems Often Have to be Circumvented by Expensive Last-Minute Fixes to Software (which then show up as software cost overruns)
- **Without Modeling Costs Statistically, There is No Way to Account for Inter-Element Correlation in the Estimating Process**



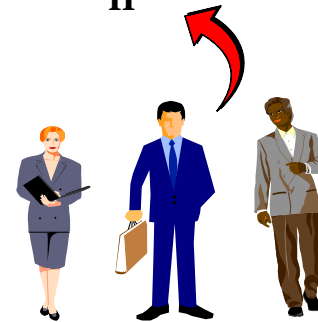
Triangular Distribution of WBS-Element Cost



Optimistic Cost
(Usually a Contractor Estimate)



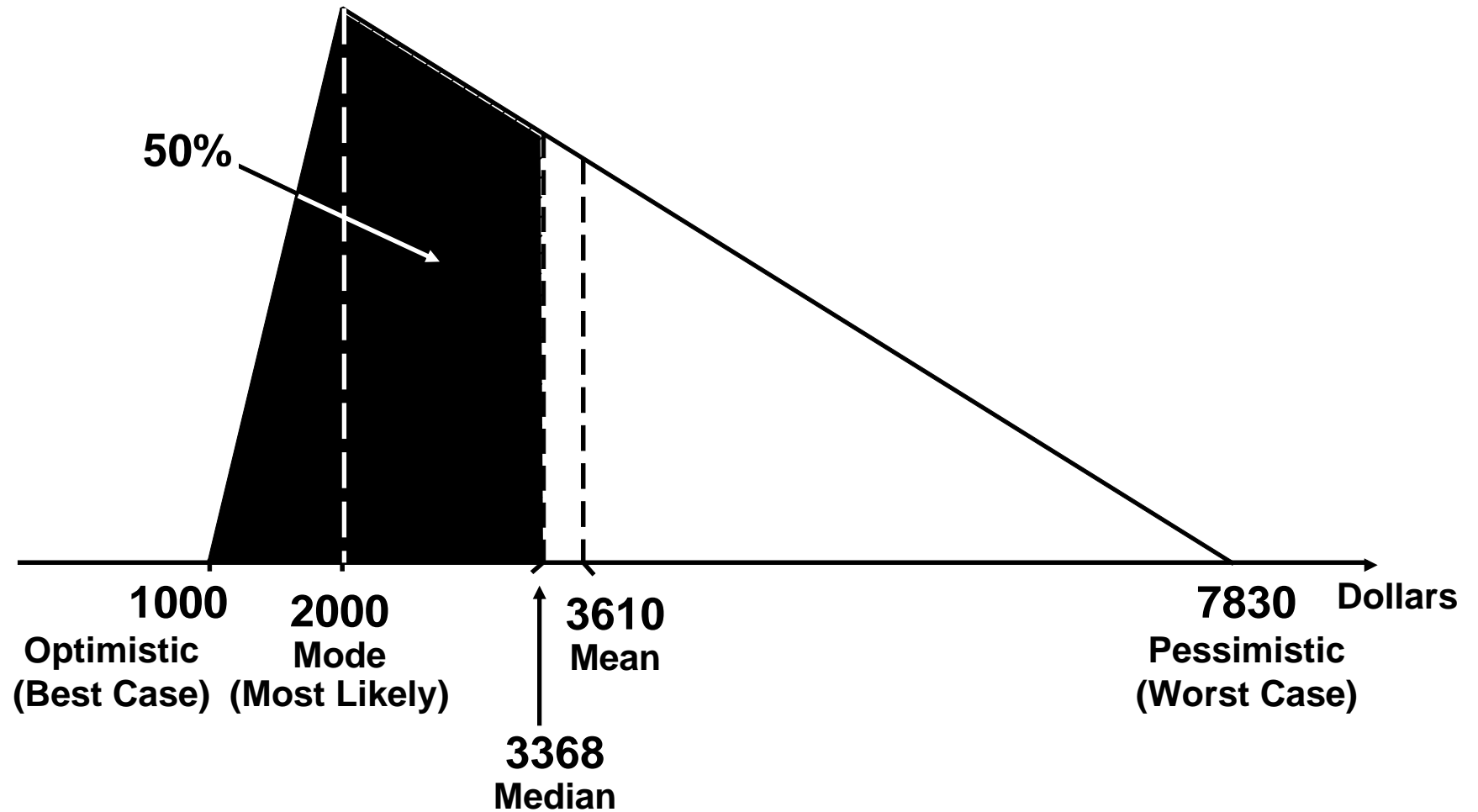
Point Estimate
(Based on Project's Technical Description)



Cost Implication of Technical, Programmatic Assessment
(Based on Project's Risk-Management Plan)

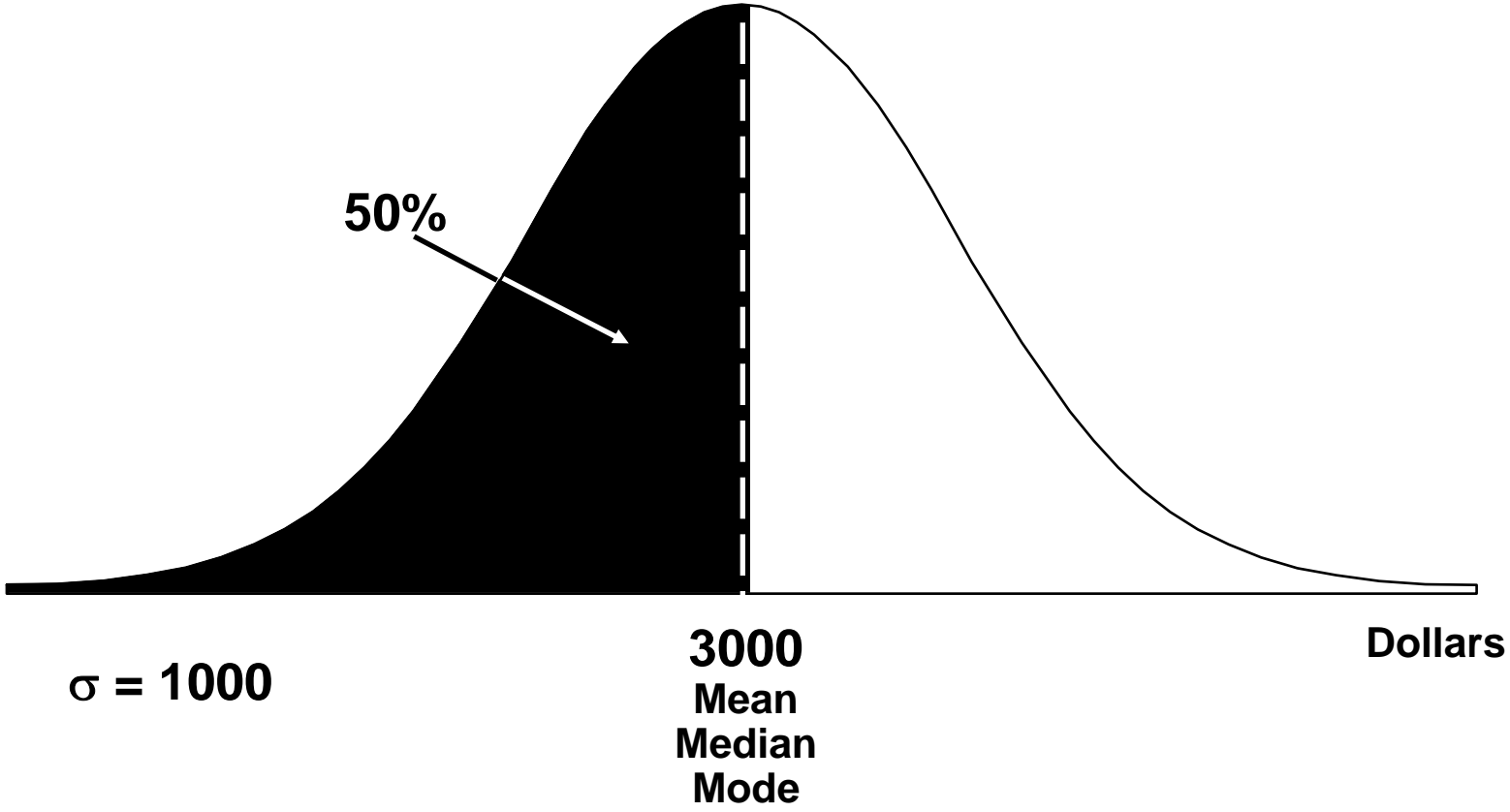


Statistical Characteristics of the Triangular Distribution



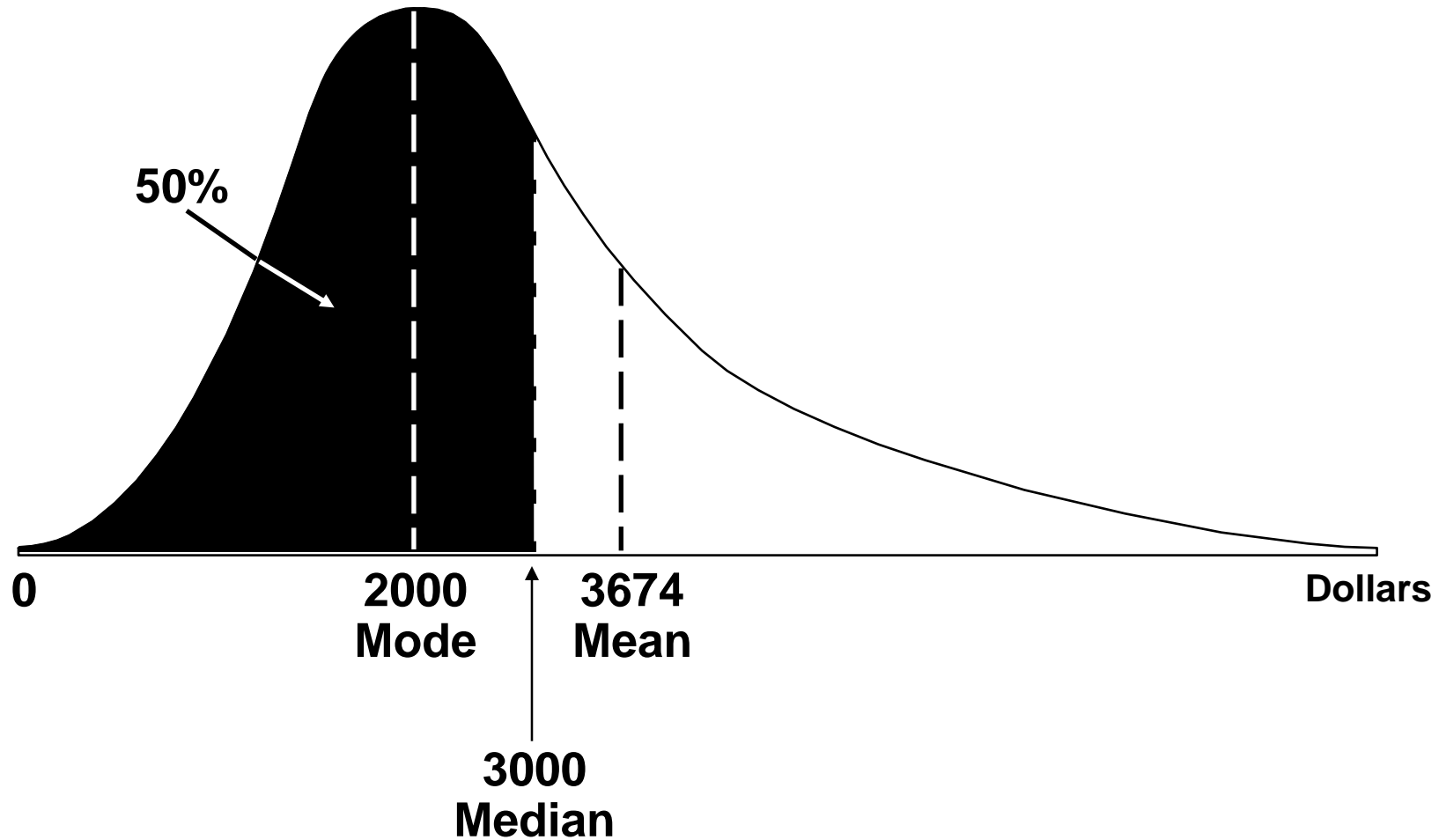


Statistical Characteristics of the Normal Distribution





Statistical Characteristics of the Lognormal Distribution





Summing Element Costs Statistically

- **Unfortunate Fact: There are No Newton's Laws or Maxwell's Equations for Cost Estimating**
- **However, There are Laws ("Theorems") of Mathematical Statistics that Apply to Portions of the Cost-Estimating Process**
- **One of these Laws is the "*Central Limit Theorem*": "If the Number of WBS Elements is 'Large,' then the Distribution of Total Cost is Approximately Normal"**
- **Another is that "The Sum of the Means is the Mean of the Sum"**



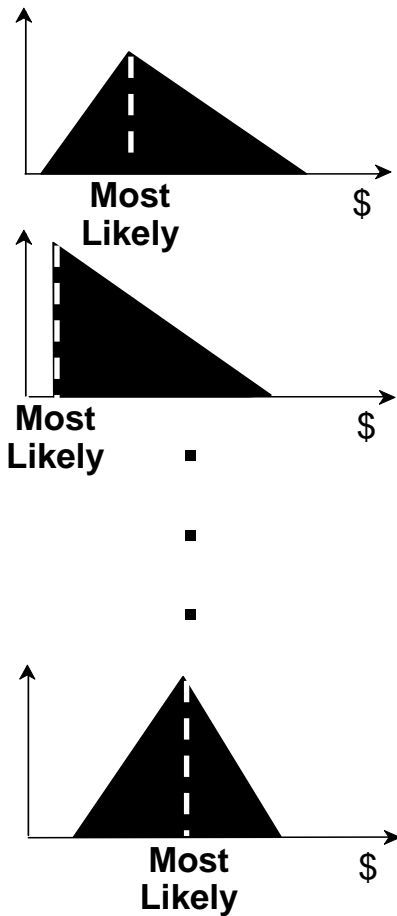
Applying the Central Limit Theorem

- **Because the Total-Cost Distribution is the Normal Distribution, the Total-Cost Mean = Total-Cost Median = Total-Cost Mode**
- **But Total-Cost Mean = Sum of WBS-Element Means**
- **Therefore ...**
 - Total-Cost Median = Sum of WBS-Element Means
 - Total-Cost Mode = Sum of WBS-Element Means
- **It Inexorably Follows that ...**
 - Sum of WBS-Element Modes < Total Cost Mode
 - Sum of WBS-Element Medians < Total Cost Median
 - Sum of WBS-Element Means = Total Cost Mean

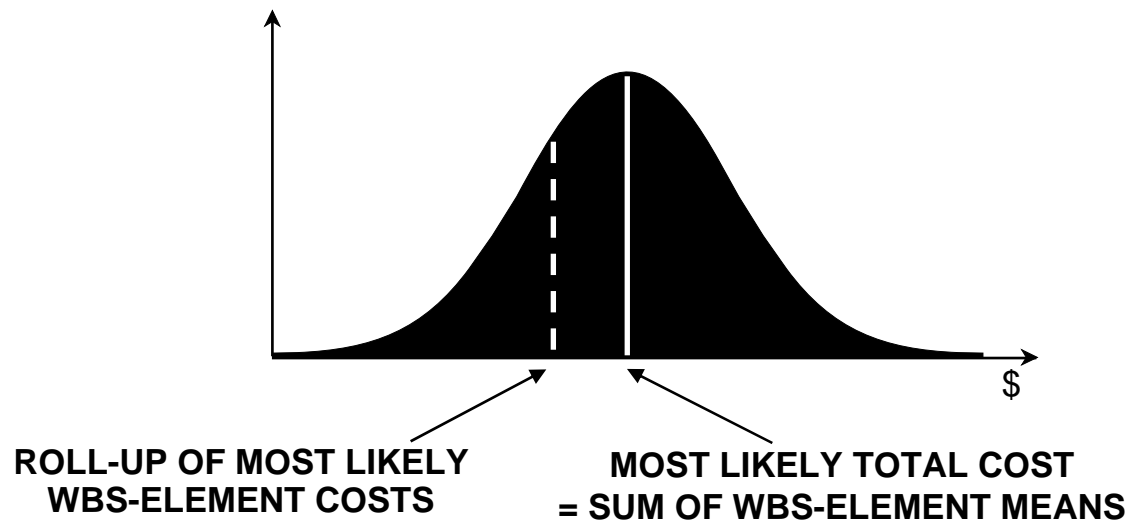


A Picture of the Central Limit Theorem

WBS-ELEMENT TRIANGULAR COST DISTRIBUTIONS



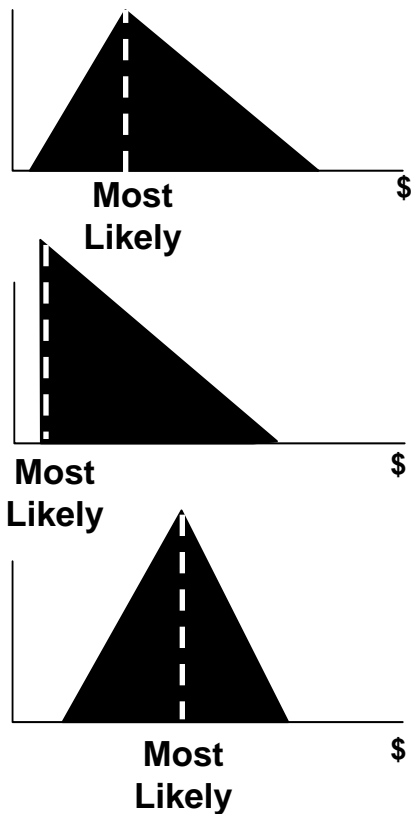
MERGE WBS-ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST NORMAL DISTRIBUTION



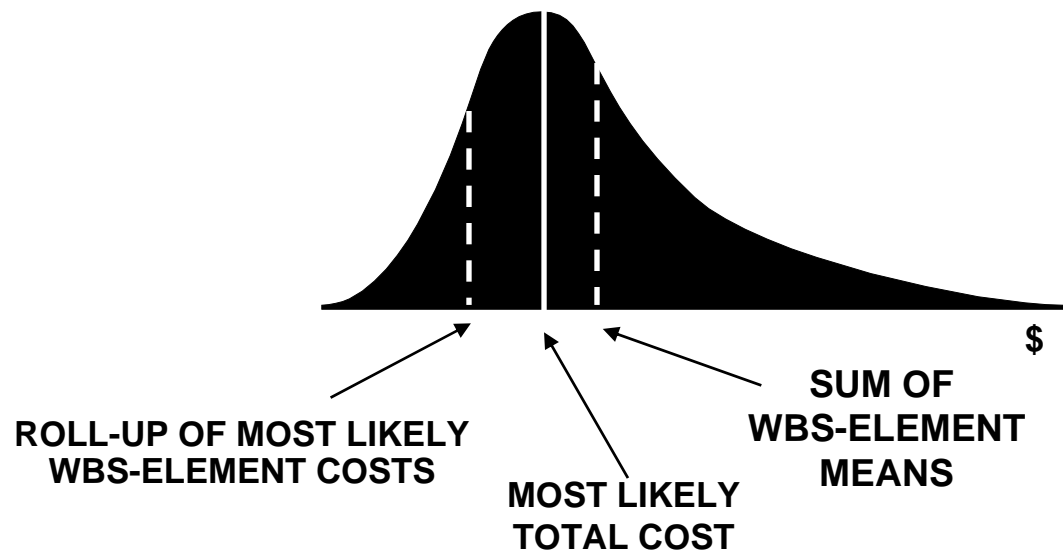


When WBS Elements Are Few...

WBS-ELEMENT TRIANGULAR COST DISTRIBUTIONS



MERGE WBS-ELEMENT COST DISTRIBUTIONS INTO TOTAL-COST LOGNORMAL DISTRIBUTION





Alternatively, We Can Apply Monte Carlo Simulation

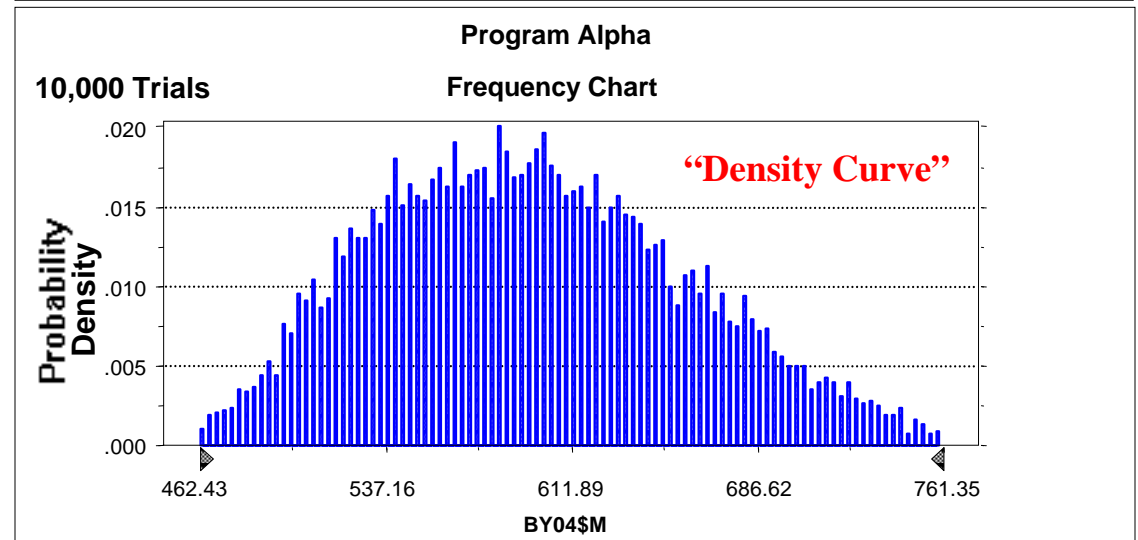
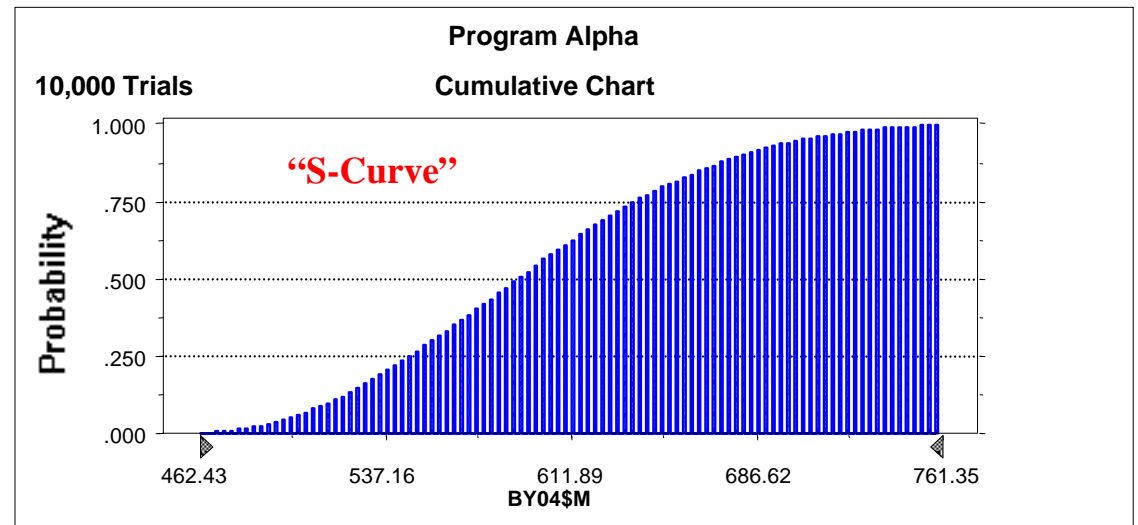
- In Engineering, ***Computer Simulation of Mission Operations*** is Standard Practice, with Key Characteristics Modeled by Monte Carlo Analysis of Random Variables, e.g.,
 - Pointing Accuracy
 - Data Throughput
 - Retro Rocket Thrust
 - Decision Timing
- Cost-Risk Analysis Enables the Cost Analyst to Conduct a ***Computer Simulation of Cost***
 - WBS-Element Costs Are Modeled As Random Variables
 - Total Cost Distribution is Established by Monte Carlo Simulation of the Sum of the WBS-Element Cost Probability Distributions



What a Cost Estimate Looks Like

| <u>Percentile</u> | <u>Value</u> |
|-------------------|--------------|
| 10% | 516.81 |
| 20% | 538.98 |
| 30% | 557.85 |
| 40% | 575.48 |
| 50% | 592.72 |
| 60% | 609.70 |
| 70% | 629.19 |
| 80% | 650.97 |
| 90% | 683.01 |

| <u>Statistics</u> | <u>Value</u> |
|--------------------|--------------|
| Trials | 10,000 |
| Mean | 596.40 |
| Median | 592.72 |
| Mode | -- |
| Standard Deviation | 63.18 |
| Range Minimum | 450.19 |
| Range Maximum | 796.68 |





Application of Statistical Laws

- **Both the Normal and Lognormal Distributions are “Two-Parameter” Distributions**
- **Full Knowledge of the Distribution, Including its S-Curve, is Contained in Any Pair of Independent Numerical Characteristics of the Distribution**
 - Mean and Standard Deviation (“sigma” value)
 - Median and Mean-Absolute-Deviation
 - 50th Percentile and 70th Percentile
- **The Mean and Standard Deviation of the Total-Cost Distribution Can be Derived From the Means, Standard Deviations, and Correlations of the Individual WBS-Element Distributions**
- **All Percentiles of the Total-Cost Distribution Can be Calculated from the Total-Cost Mean and Standard Deviation**



Graphical Representations of Probability Distributions

- **Density Curve**

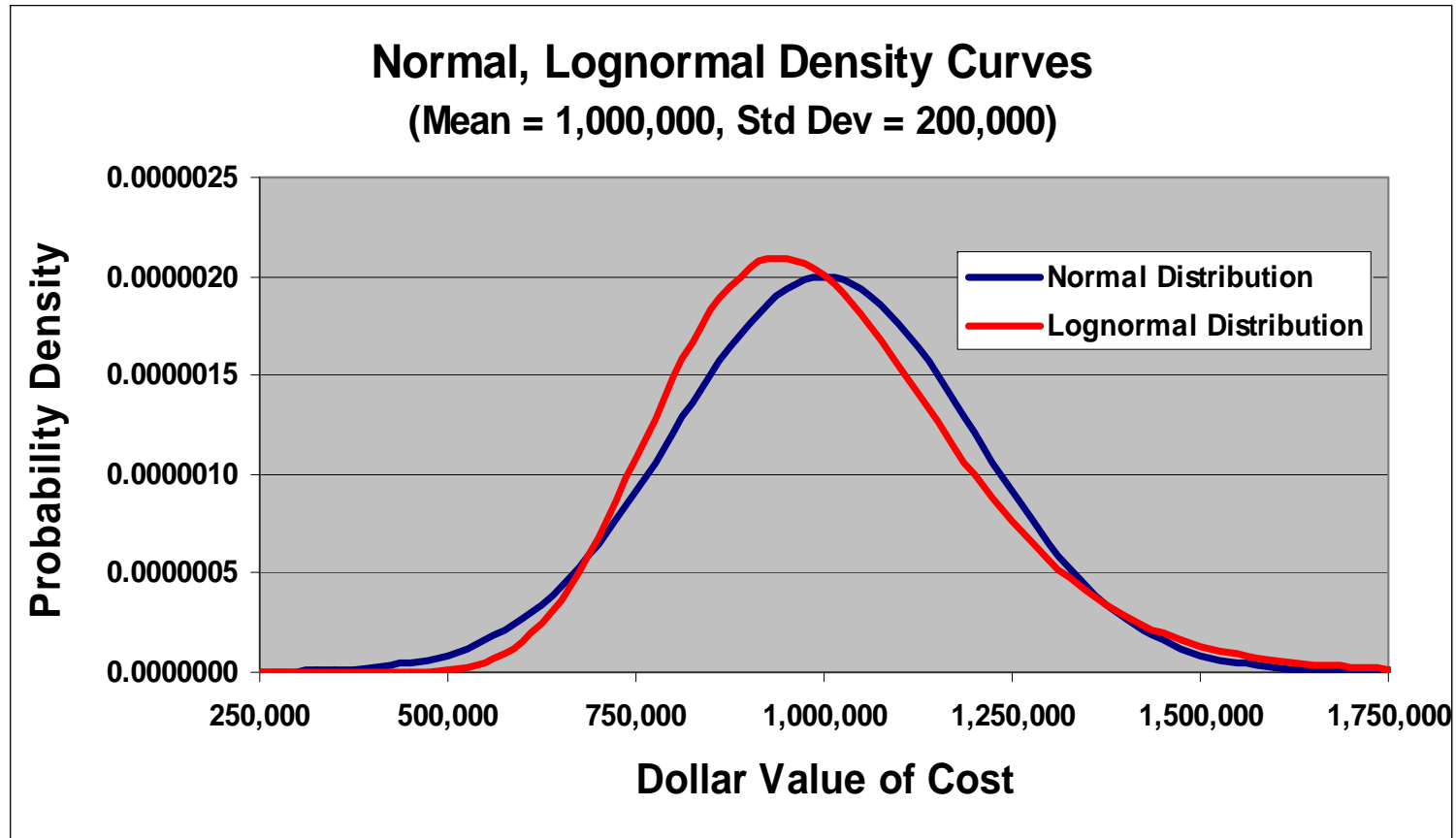
- Graph of the Probability Density Function
- Area Under the Curve is 1
- Height is 0 at Both Ends
- Highlights the Center (Mean, Median, Mode) of the Distribution
- Illustrates its Spread (Standard Deviation)

- **S-Curve**

- Graph of the Cumulative Distribution Function
- Rises from 0 to 1 on the Vertical Axis
- Slope is Positive at Every Point (if curve is continuous)
- Displays the Percentiles of the Distribution, i.e., for Each Possible Dollar Value of Budgeted Funding, the Probability of Avoiding a Cost Overrun Can be Read from the Graph



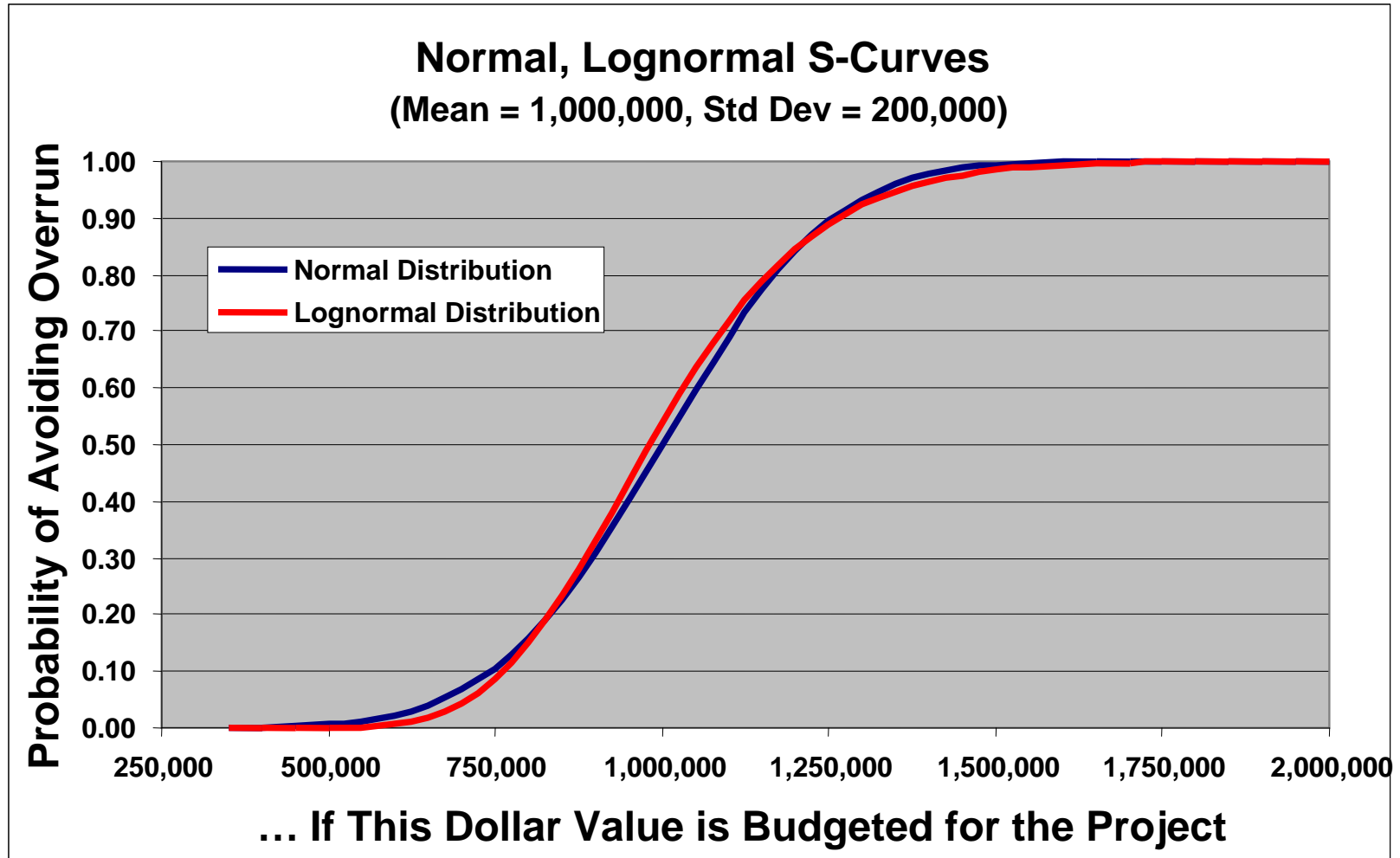
Density Curves Constructed from Known Mean and Standard Deviation



Note: Normal Distribution is Symmetric, but Lognormal is Skewed.



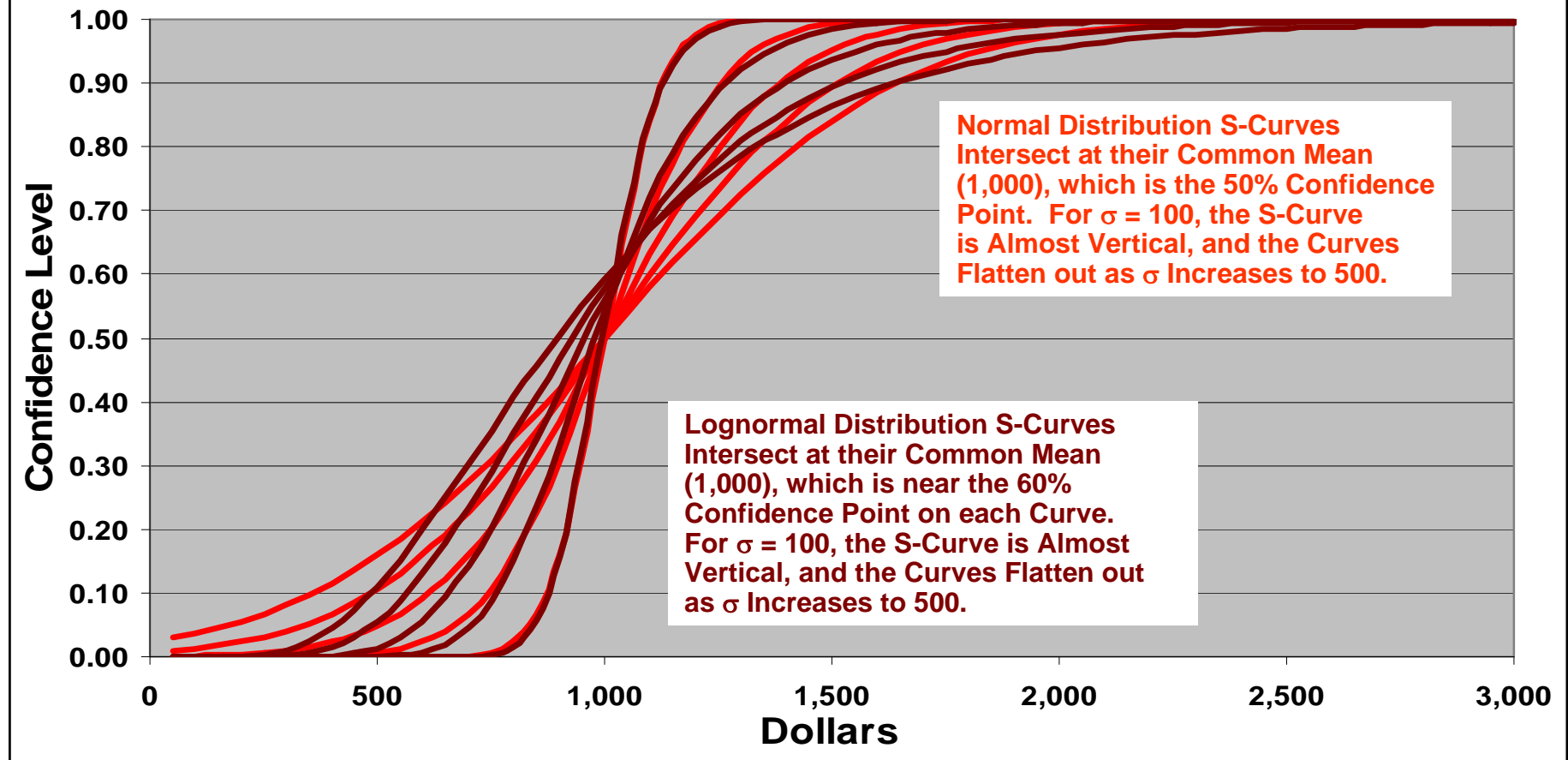
S-Curves Constructed from Known Mean and Standard Deviation





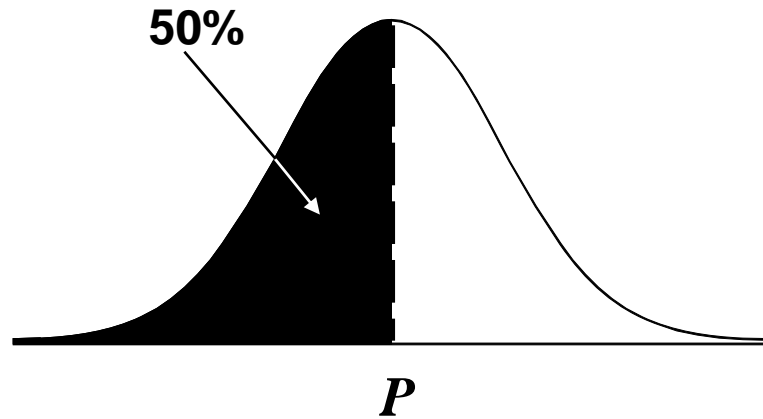
More Uncertainty Means a Flatter S-Curve

S-Curves for Normal and Lognormal Distributions with Means = 1,000 and Increasing Sigma Values





How the Normal and Lognormal Distributions are Related

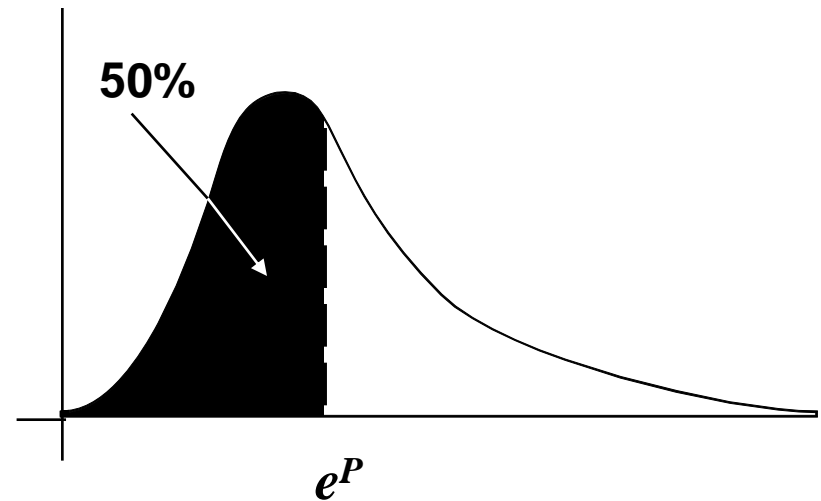
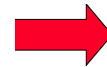


X

Normal Distribution

$P = \text{Mean} = \text{Median} = \text{Mode}$

$Q = \text{Standard Deviation}$



$Y = e^X$

Lognormal Distribution

$\text{Mode} = e^{P-Q^2}$

$\text{Median} = e^P$

$\mu = \text{Mean} = e^{P+1/2Q^2}$

$\sigma = \text{Standard Deviation}$



Normal, Lognormal Percentiles, Based on Mean and Standard Deviation

Table from Back of Your Statistics Textbook

| PERCENTILE | $1 - \alpha$ | z_α |
|------------|--------------|------------|
| 95 | .95 | 1.64485 |
| 90 | .90 | 1.28155 |
| 80 | .80 | 0.84162 |
| 70 | .70 | 0.52440 |
| 60 | .60 | 0.25335 |
| 50 | .50 | 0.00000 |
| 40 | .40 | -0.25335 |
| 30 | .30 | -0.52440 |
| 20 | .20 | -0.84162 |
| 10 | .10 | -1.28155 |
| 5 | .05 | -1.64485 |

Explicit Expressions Based on Relationship between the Normal and Lognormal Distributions:

$$\text{Normal Percentile} = P + z_\alpha Q$$

$$\text{Lognormal Percentile} = \exp\{P + z_\alpha Q\},$$

$$\text{where } P = 0.5 \ln[\mu^4 / (\mu^2 + \sigma^2)]$$

$$\text{and } Q = \{\ln[1 + (\sigma^2 / \mu^2)]\}^{1/2}$$

| Mean = | 1,000,000 | |
|------------|-----------|-----------|
| Std Dev = | 200,000 | |
| Percentile | Normal | Lognormal |
| 5th | 671,029 | 707,965 |
| 10th | 743,690 | 760,780 |
| 15th | 792,713 | 798,622 |
| 20th | 831,676 | 830,035 |
| 25th | 865,102 | 857,969 |
| 30th | 895,120 | 883,854 |
| 35th | 922,936 | 908,537 |
| 40th | 949,331 | 932,595 |
| 45th | 974,868 | 956,479 |
| 50th | 1,000,000 | 980,581 |
| 55th | 1,025,132 | 1,005,290 |
| 60th | 1,050,669 | 1,031,035 |
| 65th | 1,077,064 | 1,058,338 |
| 70th | 1,104,880 | 1,087,893 |
| 75th | 1,134,898 | 1,120,715 |
| 80th | 1,168,324 | 1,158,431 |
| 85th | 1,207,287 | 1,203,997 |
| 90th | 1,256,310 | 1,263,886 |
| 95th | 1,328,971 | 1,358,173 |



System X WBS-Element Cost Distributions

| WBS Element | L | M | H | Mean* | Sigma |
|--------------------------|-----|-----|------|-------------|--------|
| 1. Antenna | 191 | 380 | 1151 | 574 | 207.62 |
| 2. Electronics | 96 | 192 | 582 | 290 | 105.08 |
| 3. Platform | 33 | 76 | 143 | 84 | 22.63 |
| 4. Facilities | 9 | 18 | 27 | 18 | 3.67 |
| 5. Power Distribution | 77 | 154 | 465 | 232 | 83.86 |
| 6. Computers | 30 | 58 | 86 | 58 | 11.43 |
| 7. Environmental Control | 11 | 22 | 66 | 33 | 11.88 |
| 8. Communications | 58 | 120 | 182 | 120 | 25.31 |
| 9. Software | 120 | 230 | 691 | 347 | 123.68 |
| SUMS | | | | 1756 | |

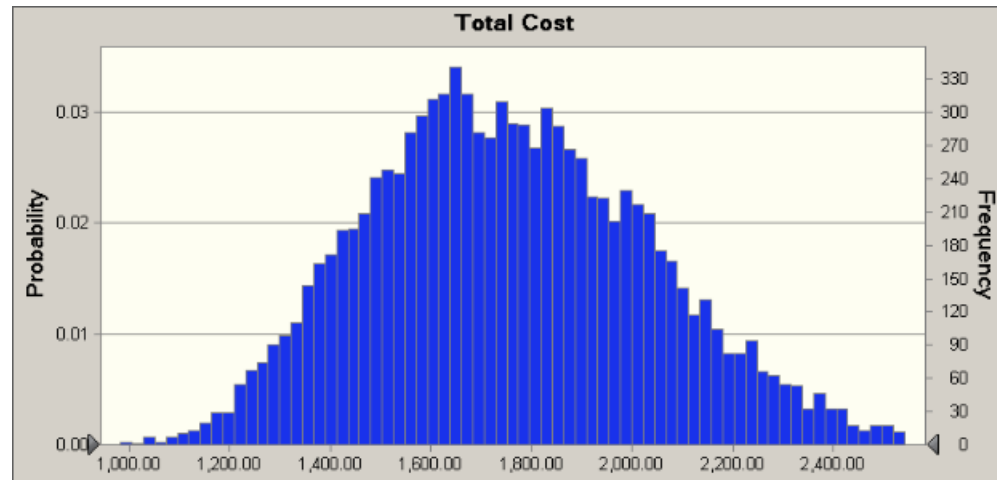
* “Mean” = Expected Value (Note: Sum of WBS-Element Means Is Equal to the Total-Cost Mean.)



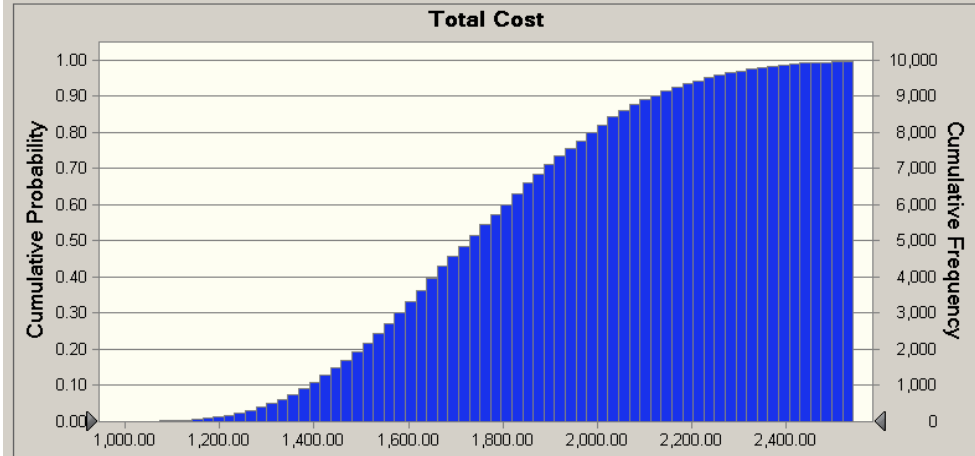
Total-Cost System X Distribution by Monte Carlo Simulation*

| Percentiles | Total Cost |
|-------------|------------|
| 5% | 1,320.16 |
| 10% | 1,399.64 |
| 15% | 1,460.09 |
| 20% | 1,508.89 |
| 25% | 1,554.15 |
| 30% | 1,592.97 |
| 35% | 1,629.75 |
| 40% | 1,662.51 |
| 45% | 1,700.33 |
| 50% | 1,739.86 |
| 55% | 1,777.18 |
| 60% | 1,818.64 |
| 65% | 1,857.85 |
| 70% | 1,898.94 |
| 75% | 1,947.32 |
| 80% | 1,999.56 |
| 85% | 2,054.81 |
| 90% | 2,130.96 |
| 95% | 2,244.74 |

| Simulation Statistics | |
|-----------------------|-----------|
| Trials | 10,000 |
| Mean | 1,756.00 |
| Median | 1,739.86 |
| Std Dev | 280.39 |
| Variance | 78,615.79 |
| Skewness | 0.2689 |
| Coef of Var | 0.1597 |
| Minimum | 983.67 |
| Maximum | 2,783.98 |
| Range | 1,800.31 |



10,000 Trials Cumulative Frequency View 9,970 Displayed



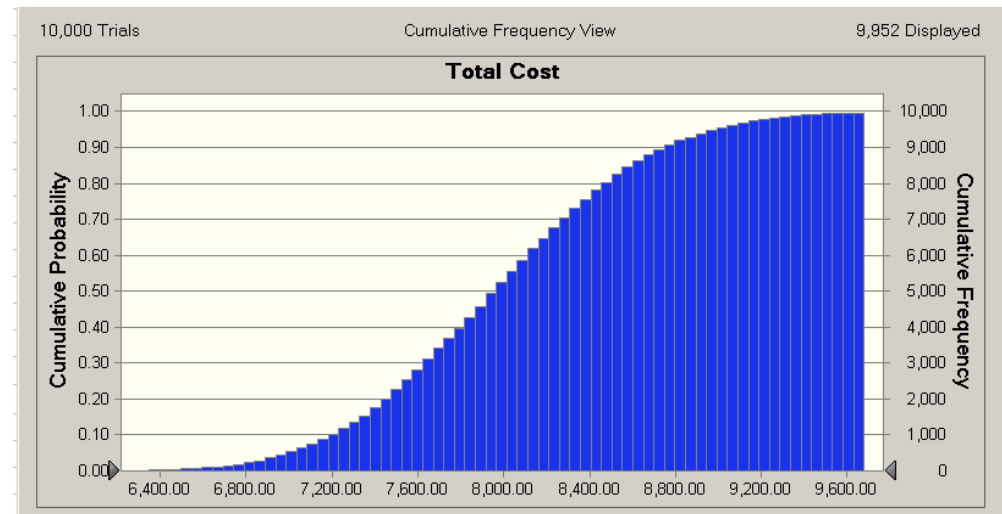
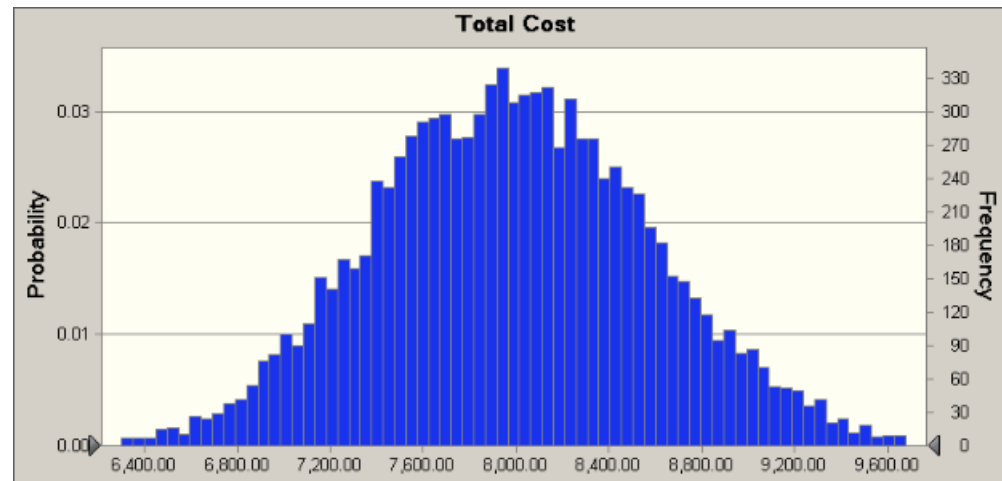
* with Inter-Element correlations assumed to be zero.



40-Element WBS Total-Cost Distribution

| Percentiles | Total Cost |
|-------------|------------|
| 5% | 7,019.93 |
| 10% | 7,223.30 |
| 15% | 7,370.02 |
| 20% | 7,476.65 |
| 25% | 7,568.07 |
| 30% | 7,652.51 |
| 35% | 7,736.33 |
| 40% | 7,823.36 |
| 45% | 7,903.70 |
| 50% | 7,976.49 |
| 55% | 8,055.76 |
| 60% | 8,135.56 |
| 65% | 8,216.88 |
| 70% | 8,301.25 |
| 75% | 8,393.70 |
| 80% | 8,496.02 |
| 85% | 8,609.65 |
| 90% | 8,768.05 |
| 95% | 9,007.00 |

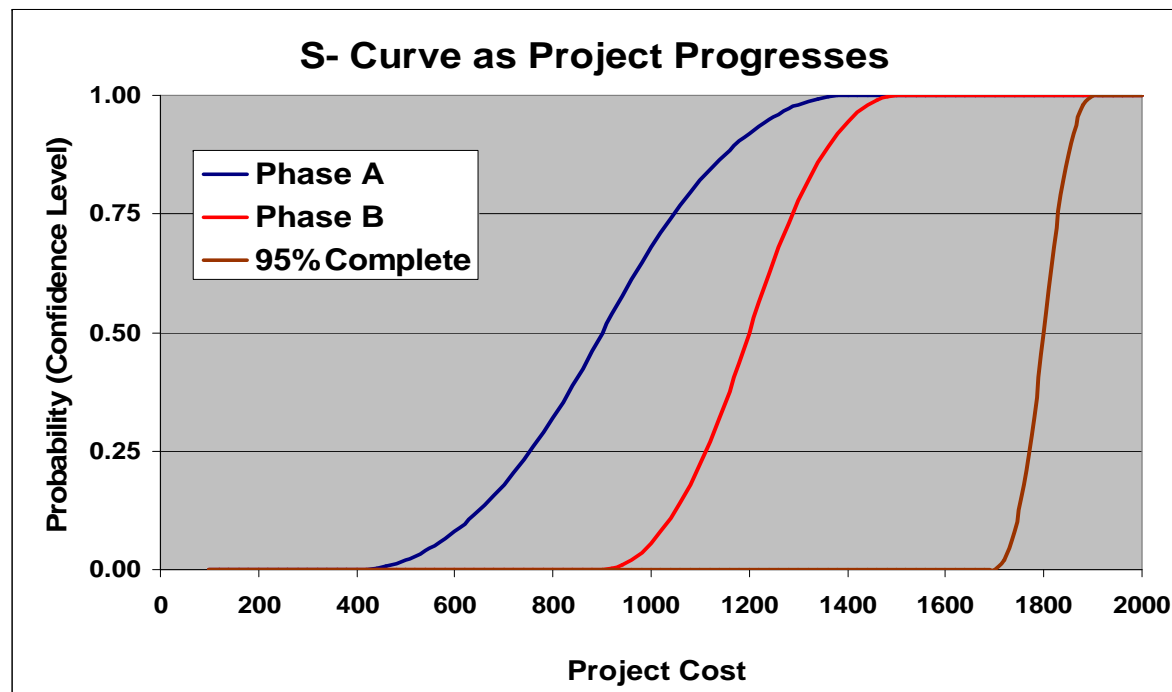
| Simulation Statistics | |
|-----------------------|------------|
| Trials | 10,000 |
| Mean | 7,990.00 |
| Median | 7,976.49 |
| Std Dev | 602.98 |
| Variance | 363,581.73 |
| Skewness | 0.1450 |
| Coef of Var | 0.0755 |
| Minimum | 5,932.13 |
| Maximum | 10,439.07 |
| Range | 4,506.95 |





S-Curves Based on Estimates at Successive Project Phases

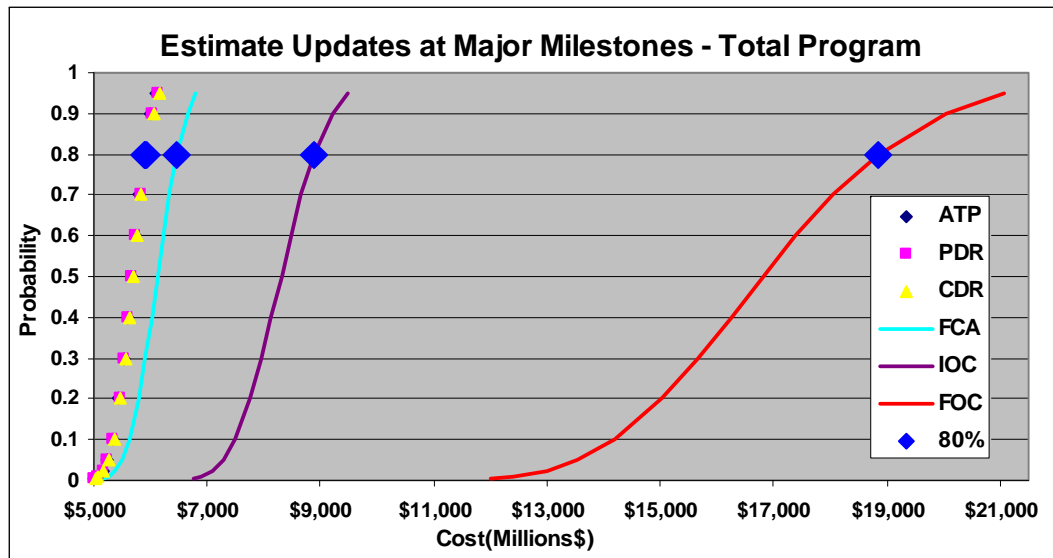
- **Folklore Says that S-Curve Starts Out Somewhat Flattish on Left Side of Time Chart and Moves to Right and Straightens Up As Time Passes**
- **However, Up to This Time, No Convincing Data-based Analysis Has Been Reported of This Phenomenon (to My Knowledge)**





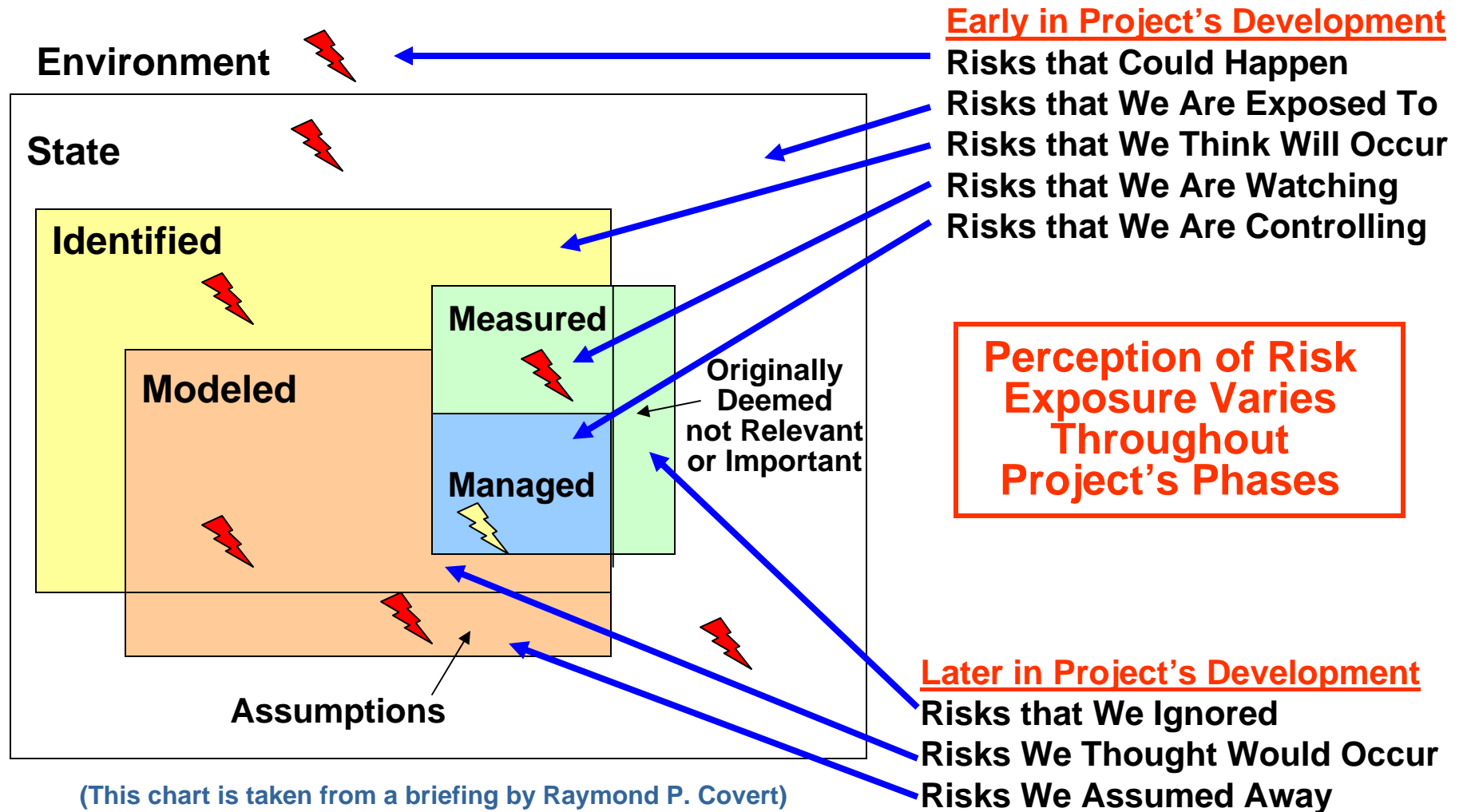
A Word from the Opposition

- **Lack of Supporting Data Allows Alternative Theories to Arise**
- **One Alternative Theory Claims the Opposite of the Folklore Theory**
 - **S-curve is Narrower at Beginning of Project When Risks are Not Well Understood or Their Existence is Denied**
 - **As Project Progresses, Some Risks are Uncovered, So S-curve Widens Somewhat**
 - **When Project Reaches Its Most Intense Stage, All Risks Have Manifested Themselves, and the S-curve is at its Widest**





What the Opposition is Thinking



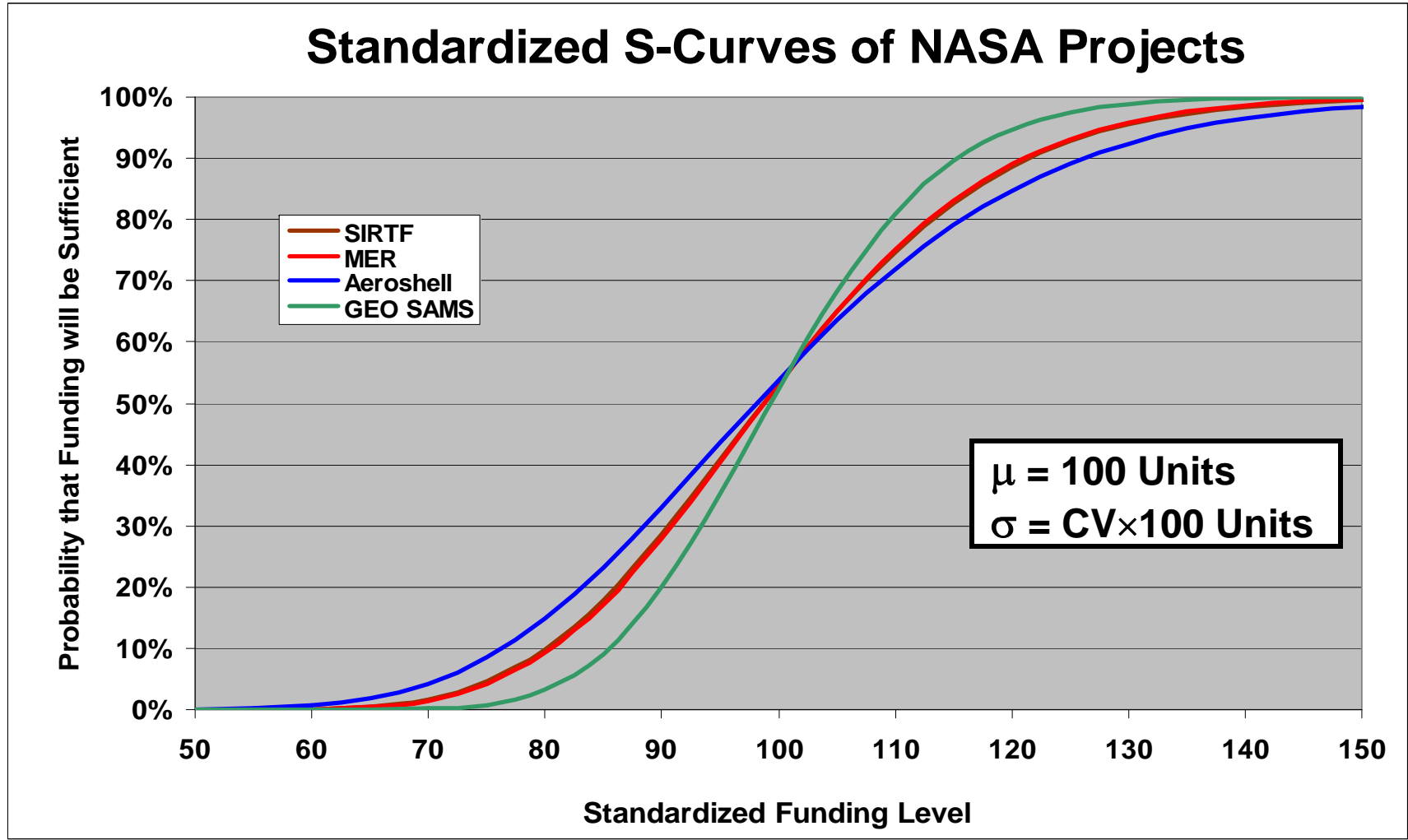


Which Model of S-Curve Behavior is Correct?

- **Unfortunately, We Don't Know the Answer at This Point – Expressing Cost Estimates as S-Curves is a Relatively Recent Phenomenon**
 - DoD Started Doing so Routinely only about 15 Years Ago
 - NASA Much Later, about 5 Years Ago
- **However, NASA's IPAO Has a Research Project in Place to Provide the Answer**
 - Records were Compiled of NASA Contractor, Project, and IPAO Estimates in the Form of S-Curves or Equivalent Statistical Descriptions of the Estimated Cost at Various Project Phases
 - Representation of Cost S-Curves in “Standardized Lognormal” Form Protects Sensitive Data from Public Release
 - So Far, Results of the Study are Inconclusive for Various Reasons



NASA S-Curves in “Standardized Lognormal” Form





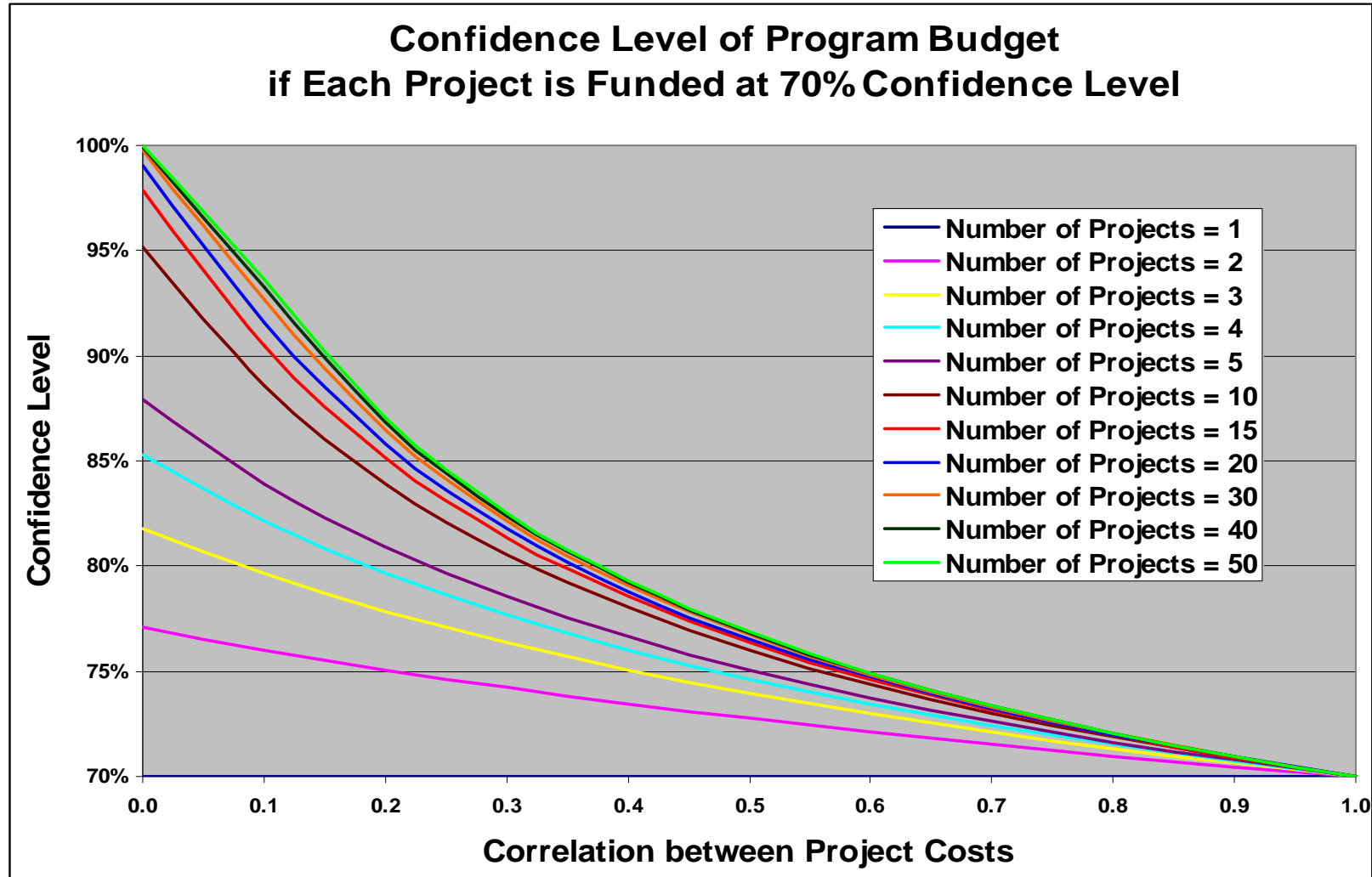
Confidence Level of Program Budget if Each Project is Funded at 70% Confidence Level

| Budget Each Project at Confidence Level 70% | Then This is the Confidence Level of Program Budget if the Number of Projects is: | | | | | | | | | | |
|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 1 | 2 | 3 | 4 | 5 | 10 | 15 | 20 | 30 | 40 | 50 |
| Correlation = 0.0 (Uncorrelated) | 70.00% | 77.08% | 81.81% | 85.29% | 87.95% | 95.14% | 97.89% | 99.05% | 99.80% | 99.95% | 99.99% |
| Correlation = 0.1 | 70.00% | 76.02% | 79.65% | 82.12% | 83.92% | 88.55% | 90.51% | 91.58% | 92.71% | 93.30% | 93.66% |
| Correlation = 0.2 | 70.00% | 75.08% | 77.87% | 79.65% | 80.89% | 83.92% | 85.13% | 85.78% | 86.47% | 86.82% | 87.04% |
| Correlation = 0.3 | 70.00% | 74.23% | 76.36% | 77.66% | 78.54% | 80.57% | 81.34% | 81.75% | 82.18% | 82.40% | 82.53% |
| Correlation = 0.4 | 70.00% | 73.46% | 75.08% | 76.02% | 76.65% | 78.03% | 78.54% | 78.81% | 79.08% | 79.22% | 79.30% |
| Correlation = 0.5 | 70.00% | 72.76% | 73.96% | 74.64% | 75.08% | 76.02% | 76.36% | 76.54% | 76.72% | 76.81% | 76.86% |
| Correlation = 0.6 | 70.00% | 72.12% | 72.99% | 73.46% | 73.76% | 74.39% | 74.62% | 74.73% | 74.84% | 74.90% | 74.94% |
| Correlation = 0.7 | 70.00% | 71.53% | 72.12% | 72.43% | 72.63% | 73.03% | 73.17% | 73.24% | 73.31% | 73.35% | 73.37% |
| Correlation = 0.8 | 70.00% | 70.98% | 71.34% | 71.53% | 71.64% | 71.87% | 71.95% | 71.99% | 72.03% | 72.05% | 72.07% |
| Correlation = 0.9 | 70.00% | 70.47% | 70.64% | 70.72% | 70.77% | 70.87% | 70.91% | 70.93% | 70.94% | 70.95% | 70.96% |
| Correlation = 1.0 | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% | 70.00% |

- **This is a “Rule of Thumb” Table – Underlying the Numbers are Assumptions that Allow Computational Simplicity**
 - All Projects Have the Same Cost Probability Distribution
 - Costs of All Projects are Normally Distributed
- **In any Actual Scenario, these Assumptions Will Not be Valid, but, Given Specifics of the Scenario, i.e., Project Cost Distributions and Inter-Project Cost Correlations, the Correct Numbers Can be Determined by Monte Carlo Simulation**



Program Confidence-Level Graphics for Project Funding at 70% Level





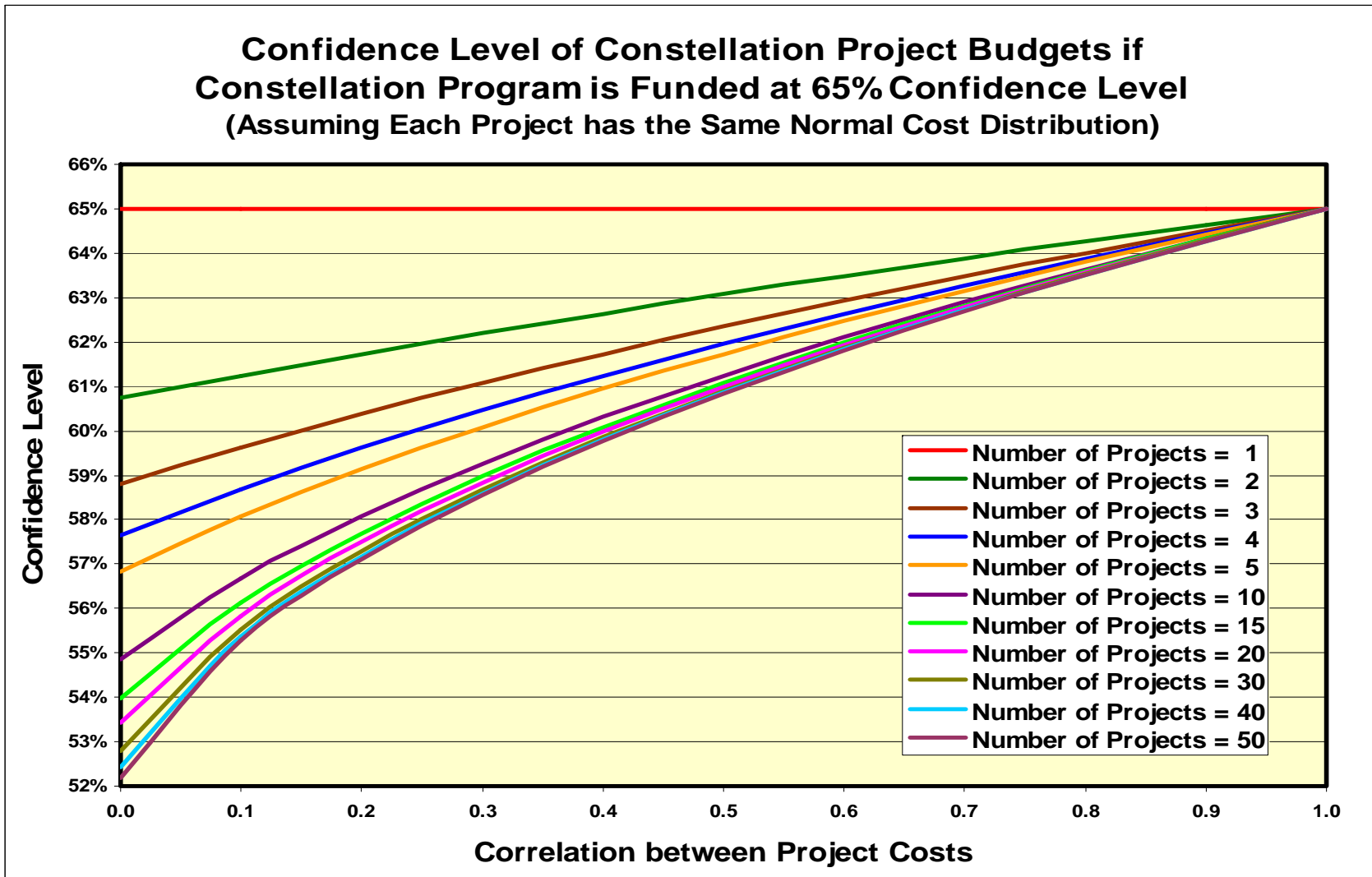
Confidence Levels of Project Budgets if Constellation Program is Funded at 65% Level

| Budget Constellation Program at Confidence Level 65% | Then This is the Appropriate Confidence Level of Each Project Budget if the Number of Projects is: | | | | | | | | | | |
|--|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 1 | 2 | 3 | 4 | 5 | 10 | 15 | 20 | 30 | 40 | 50 |
| Correlation = 0.0 (Uncorrelated) | 65.00% | 60.74% | 58.80% | 57.64% | 56.84% | 54.85% | 53.96% | 53.43% | 52.80% | 52.43% | 52.17% |
| Correlation = 0.1 | 65.00% | 61.25% | 59.63% | 58.69% | 58.08% | 56.67% | 56.12% | 55.83% | 55.52% | 55.36% | 55.27% |
| Correlation = 0.2 | 65.00% | 61.73% | 60.38% | 59.63% | 59.14% | 58.08% | 57.69% | 57.49% | 57.28% | 57.17% | 57.11% |
| Correlation = 0.3 | 65.00% | 62.20% | 61.08% | 60.47% | 60.09% | 59.27% | 58.97% | 58.82% | 58.67% | 58.59% | 58.55% |
| Correlation = 0.4 | 65.00% | 62.64% | 61.73% | 61.25% | 60.94% | 60.31% | 60.09% | 59.97% | 59.86% | 59.80% | 59.77% |
| Correlation = 0.5 | 65.00% | 63.07% | 62.35% | 61.97% | 61.73% | 61.25% | 61.08% | 61.00% | 60.91% | 60.87% | 60.84% |
| Correlation = 0.6 | 65.00% | 63.48% | 62.93% | 62.64% | 62.47% | 62.11% | 61.98% | 61.92% | 61.86% | 61.83% | 61.81% |
| Correlation = 0.7 | 65.00% | 63.88% | 63.48% | 63.28% | 63.15% | 62.90% | 62.81% | 62.77% | 62.73% | 62.71% | 62.69% |
| Correlation = 0.8 | 65.00% | 64.26% | 64.01% | 63.88% | 63.80% | 63.64% | 63.59% | 63.56% | 63.54% | 63.52% | 63.51% |
| Correlation = 0.9 | 65.00% | 64.64% | 64.51% | 64.45% | 64.42% | 64.34% | 64.32% | 64.30% | 64.29% | 64.28% | 64.28% |
| Correlation = 1.0 | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% | 65.00% |

- **Again, This is a “Rule of Thumb” Table – Underlying the Numbers are Assumptions that Allow Computational Simplicity**
 - All Projects Have the Same Cost Probability Distribution
 - Costs of All Projects are Normally Distributed
- **In the Case of the Constellation Program, Projects are More Likely to be Highly Correlated than Would, say, the Projects belonging to a Planetary Exploration Program, because Constellation Projects Have to “Fit” Together to Achieve Program Objectives**



Project Confidence-Level Graphics for Constellation Funding at 65% Level





A Final Note: Cost and Schedule are Related

- **If the Schedule Slips, i.e., the Project Takes Longer than Anticipated to Complete, then its Cost will Increase**
 - “Burn Rate”
 - “Standing Army” Must be Paid
- **Whenever the Schedule is Updated, the Cost Estimate Must be Updated Also**
- **Cost and Schedule are Correlated**
 - A Program with a Longer Schedule Generally has Higher Cost
 - A Program with a Short Schedule Generally has Lower Cost



Cost Risk and Schedule Risk are Related

- **The Risk “Watch List” Consists Mainly of Items that Impact Both Cost and Schedule**
- **The Risk Management Plan Involves Adjustments to Both Cost and Schedule**
- **Mathematically, the Relationship between Cost Risk and Schedule Risk Can be Modeled as the “Joint Bivariate Probability Distribution” of the Cost and Schedule Risk Distributions**
 - C = Cost (as a probability distribution)
 - S = Schedule (as a probability distribution)
 - $P\{C \leq x, S \leq y\}$ is the Joint Bivariate Probability that Cost is Less than or Equal to x Dollars and Schedule is Shorter than or Equal to y Months

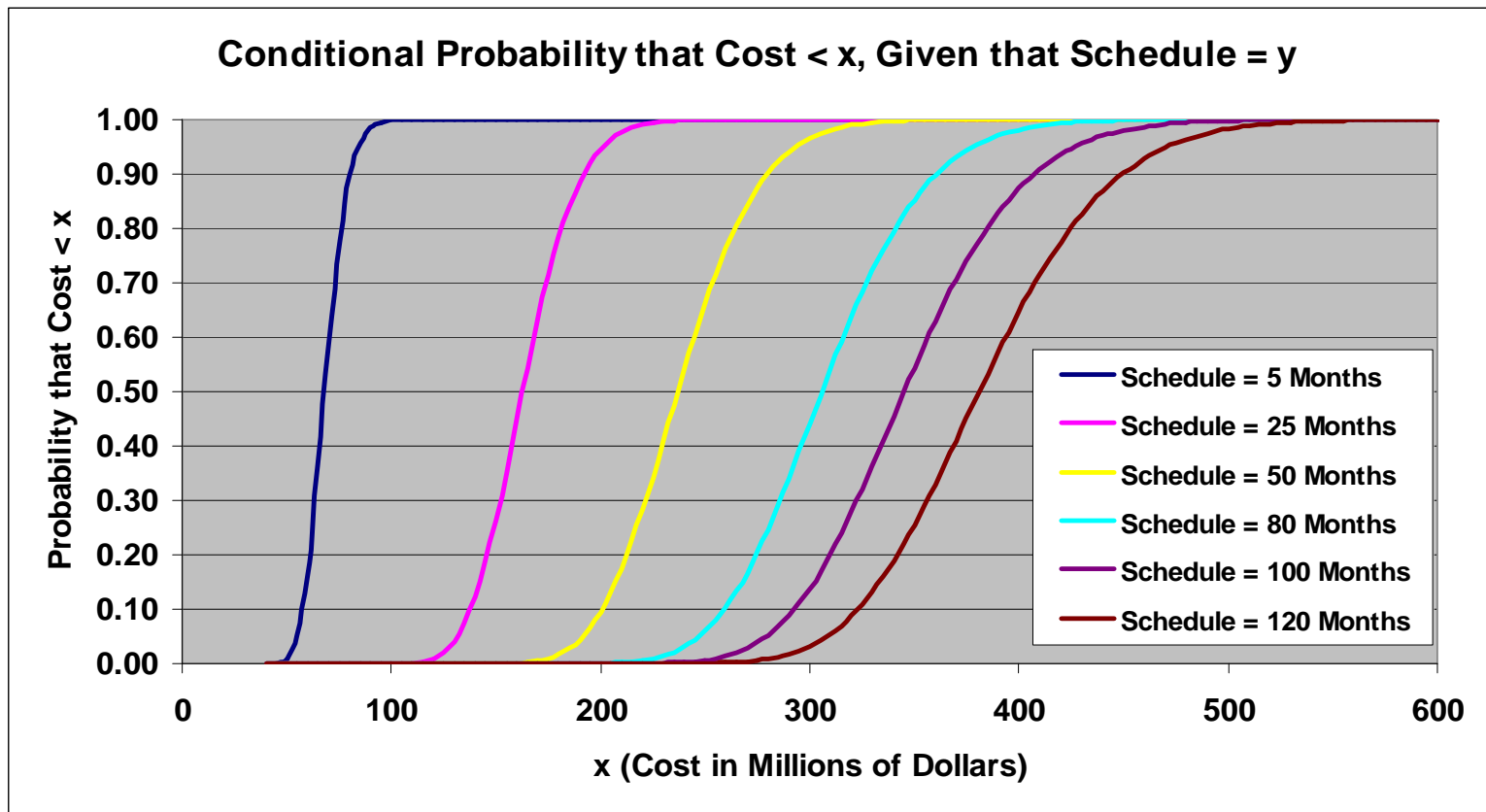


Conditional Probabilities

- **Using the Joint Bivariate Probability, We Can Calculate the Following Two Conditional Probabilities:**
 - $P\{C \leq x \mid S = y\}$, the Conditional Probability that Cost is Less than or Equal to x Dollars, Given that Schedule is Equal to y Months
 - $P\{S \leq y \mid C = x\}$, the Conditional Probability that Schedule is Shorter than or Equal to y Months, Given that Cost is Equal to x Dollars
- **If We Assume that Both Cost and Schedule are Expressed as Lognormal Distributions, then the Conditional Probability Depends on the Following Quantities**
 - Mean and Standard Deviation of Cost
 - Mean and Standard Deviation of Schedule
 - Correlation between Them



Conditional Probability Distribution of Cost, Given Schedule Duration



Transition from Monte Carlo Analysis of Cost and Schedule:

- Cost Distribution is Lognormal with $\mu = 200$, $\sigma = 30$
- Schedule Distribution is Lognormal with $\mu = 36$, $\sigma = 5$
- Correlation between Cost and Schedule is 0.5



Summary

- **S-Curves**
 - Associate Each Dollar Value of Cost with Probability that Project Can be Successfully Completed within that Budget
 - Provide Decisionmakers Insight into the Probability of Cost Overruns (and Schedule Slips)
- **The Probabilistic Component of the S-Curve Arises from Issues of Risk and Other Uncertainties in How Project is to be Pursued**
- **Instead of Newton's Laws and Maxwell's Equations, We Apply Statistical Theorems to Derive Cost S-Curves**
- **We Can Simulate Project Cost (and Schedule) Just as Engineers Simulate Project Performance**